Extended Solutions

1.1 Sets and Groups of Numbers

1A. The fraction x/y does not have to be an integer, natural number, or whole number. An example would be 2/5.

2D. On the test, if a number is not π or an imperfect square root, it is rational. Thus, 7.77 is rational. The number 7 is natural and rational. The number $7/7 = 1$ is a natural number and rational number.

3D. Choice A ($\sqrt{12}$), Choice B ($\sqrt{6}$) and Choice C ($\sqrt{18}$) are imperfect square roots and are irrational. Only Choice D ($\sqrt{36} = 6$) is rational.

4C. The fraction simplifies to $5\pi/-1\pi = 5/-1 = -5$. The number –5 is an integer and rational.

5B. An easy way to solve the problem is to plug in a number for x. We will let $x = 5$, which makes $\sqrt{5}$ irrational. Multiplying ($\sqrt{5} \bullet \sqrt{5} = 5$), subtracting ($\sqrt{5} - \sqrt{5} = 0$), dividing by itself ($\sqrt{5} \div \sqrt{5} = 1$), and raising the root to the second power ($\sqrt{5}^2 = 5$) makes the answer rational. Adding the root by itself ($\sqrt{5} + \sqrt{5} = 2\sqrt{5}$) keeps it irrational.

6C. A real number is either irrational or rational. It cannot be both.

7E. Choice A ($\sqrt{55}$) and Choice B ($\sqrt{20}$) have imperfect square roots as members which are irrational numbers. Choice C ($\sqrt{-9} = 3i$) and Choice D ($1i$) have imaginary numbers. To be rational, a number must be real number. All of choice E's members are rational ($\sqrt{4/9} = 2/3$).

8A. The number pi (π) does not go on forever in a pattern. It is not rational. It is also not an integer or a natural number.

9C. The number ($15/4 = 3.75$) is not π or an imperfect square root. It is rational. The fraction does not divide evenly, so 15/4 is not an integer or a natural number.

1.2 Factors, Multiples, Primes and Prime Factorization

1E. The factors of 24 are 1,2,3,4,6,8,12,24. Adding all the factors, you get 60.

2A. The prime factorization of $25{,}200 = 2^4 \times 3^2 \times 5^2 \times 7^1$, so $A + B + C + D = 4 + 2 + 2 + 1 = 9$.

3A. The prime factorization of $264 = 2^3 \times 3^1 \times 11^1$. The number has 3 distinct prime divisors: 2, 3 and 11.

4D. The multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144 and so on. The number 92 is not a multiple of 12.

5C. The factors of 36 are 1,2,3,4,6,9,12,18,36. The number has 9 distinct factors.

6C. The first 5 prime numbers are 2,3,5,7,11. The sum of the numbers is $2 + 3 + 5 + 7 + 11 = 28$.

7C. If A is divisible by 15 and B is divisible by 6, multiplying the numbers together gives you a number that is divisible by 90. ($A = ? \times 15$, $B = ? \times 6$, so $A \times B = ? \times 15 \times 6 = ? \times 90$.) In other words, $A \times B$ will have all the factors of 90. The only answer choice that is not a factor of 90 is 12.

8A. There are only 2 "emirps" that are bigger than 20 and less than 50. Those numbers are 31 ($31 \leftrightarrow 13$) and 37 ($37 \leftrightarrow 73$).

9D. The product $64 \times 243 = 32 \times 2 \times 81 \times 3$. Thus, $64 \times 243 = 2^5 \times 2^1 \times 3^4 \times 3^1 = 2^6 \times 3^5$.

10B. Forty is the only number that is a multiple of 4 {4,8,12,16,20,24,28,32,36,<u>40</u>…} and a multiple of 5 {5,10,15,20,25,30,35,<u>40</u>…}.

11B. To find multiples of a number, you skip count by that number. We can keep adding 6 to $(x + 2)$ until it matches one of the answer choices. The answer $(x + 2) + 6 + 6 \rightarrow (x + 14)$ is also a multiple of 6. You can also plug in a number for x to make $(x + 2)$ a multiple of 6 and then check the answers for a multiple of 6. For example, if we let $x = 10$, then $(x + 2) \rightarrow (10 + 2) = \underline{12}$. When $x = 10$, the only other choice that also changes into a multiple of 6 is B $(x + 14) \rightarrow (10 + 14) = \underline{24}$.

12A. The prime factorization of $154 = 2^1 \times 7^1 \times 11^1$. Thus, $A \times 154 = 7^8 \times 11^4 \times 2^1 \times 7^1 \times 11^1$, which equals $2^1 \times 7^9 \times 11^5$.

13A. The factors of 39 are 1,3,13,39. The 2nd largest factor is 13. The factors of 45 are 1,3,5,9,15,45. The second smallest factor is 3. The sum is $3 + 13 = 16$.

14E. *Choice I* is false. Adding two primes is not always odd $(3 + 5 = 8)$. *Choice II* is false. Adding two primes is not always even $(2 + 3 = 5)$. *Choice III* is false. Adding two primes is not always prime $(5 + 7 = 12)$. None of the answers are always true.

1.3 Greatest Common Factor (GCF) and Least Common Multiple (LCM)

1A. The GCF of 30 and 15 = 15. The LCM of 20 and 10 = 20. Therefore $x - y = 15 - 20 = -5$.

2D. *Choice A* (GCF = 3, LCM = 15), *Choice B* (GCF = 5, LCM = 15), *Choice C* (GCF = 3, LCM = 30) and *Choice E* (GCF = 3, LCM = 60) are all incorrect. Only *Choice D* has a GCF of 3 and an LCM of 45.

3A. Using factor trees, $36 = 2 \times 2 \times 3 \times 3$ and $42 = 2 \times 3 \times 7$. The number $90 = 2 \times 3 \times 3 \times 5$. One 2 and one 3 can be crossed off all the trees. The GCF $= 2 \times 3 = 6$.

4D. The denominators are 15, 25, and 10. Finding the LCM using prime factors, $15 = 3^1 \times 5^1$. In addition, $25 = 5^2$ and $10 = 2^1 \times 5^1$. The LCM $= 2^1 \times 3^1 \times 5^2 = 150$. The LCD is 150.

5C. We are looking for the GCF of 1360 and 1480. Using factor trees, $1360 = 2 \times 2 \times 2 \times 2 \times 5 \times 17$ and $1480 = 2 \times 2 \times 2 \times 5 \times 37$. Three 2s and one 5 can be crossed off the trees. The GCF $= 2 \times 2 \times 2 \times 5 = 40$.

6B. We need to find the LCM of the times at which the prizes are given out. The LCM of 12, 15 and 40 = 120. All the prizes will be given out at the same time 120 minutes or 2 hours later, which will be 10:00am.

7A. Using factor trees, $56 = 2 \times 2 \times 2 \times 7$. Also, $72 = 2 \times 2 \times 2 \times 3 \times 3$ and $75 = 3 \times 5 \times 5$. There is no prime factor that is on all three trees at the same time. The GCF = 1. (1 divides evenly into all numbers.)

8E. *Choice A* (GCF = 4, LCM = 48), *Choice B* (GCF = 6, LCM = 36), *Choice C* (GCF = 2, LCM = 144) and *Choice D* (GCF = 8, LCM = 48) are all incorrect. Only *Choice E* has a GCF of 6 and a LCM of 72.

9D. The denominators are 14, 12, and 10. Finding the LCM using prime factors, $14 = 2^1 \times 7^1$. In addition, $12 = 2^2 \times 3^1$ and $10 = 2^1 \times 5^1$. The LCM $= 2^2 \times 3^1 \times 5^1 \times 7^1 = 420$. The LCD is 420.

10B. Two numbers are coprime if their GCF is 1. Choice A (GCF = 3), Choice C (GCF = 2), Choice D (GCF = 7) and choice E (GCF = 3) are all incorrect. Only Choice B has a GCF = 1 and are coprime.

11A. The denominators are 6, 15, and 75. Finding the LCM using prime factors, $6 = 2^1 \times 3^1$. In addition, $15 = 3^1 \times 5^1$ and $75 = 3^1 \times 5^2$. The LCM $= 2^1 \times 3^1 \times 5^2 = 150$. The LCD is 150.

1.4 Fractions and Mixed Numbers

1B. The factors of 24 are 1,2,3,4,6,8,12,24. The factors of 36 are 1,2,3,4,6,9,12,18,36. We are trying to make fractions that equal 5/10 or 1/2. Four such fractions can be made (1/2, 2/4, 3/6 and 6/12).

2A. The largest fraction will have the biggest numerator (biggest x) and the smallest denominator (smallest y). The biggest value of x is 15 and the smallest value of y is 7. The largest value of the fraction is $\frac{15-3}{7+5} = \frac{12}{12} = 1$.

3C. *Choice A* is true. It shows adding fractions with the same denominator. *Choice B* is true: $1 + \frac{y}{w} = \frac{w}{w} + \frac{y}{w} = \frac{w+y}{w}$. *Choice D* is true: $\frac{w}{y} - 1 = \frac{w}{y} - \frac{y}{y} = \frac{w-y}{y}$. *Choice E* is true. You can cancel the w's and get just y. *Choice C* is the only answer that is false. If this all seems confusing, just make up your own numbers for w, y and z. For example, if you let $w = 1$, $y = 2$ and $z = 3$, all the choices will come out true except *Choice C*. ($\frac{1+2}{1} = \frac{3}{1} = 3$. That answer is not equal to y, which is 2.)

4D. In the problem, they are looking for the fraction that is halfway between 1/2 and 7/8. First, add the fractions: $1/2 + 7/8 = 4/8 + 7/8 = 11/8$. Now divide the answer by two: $11/8 \div 2 = 11/8 \div 2/1 = 11/8 \times 1/2 = 11/16$.

5A. To solve the problem, we can turn the top and bottom into a single fraction and divide.
TOP: $1/2 + 1/3 = 3/6 + 2/6 = 5/6$
BOTTOM: $2/1 - 1/4 = 8/4 - 1/4 = 7/4$
Now divide the top fraction and bottom fraction: $5/6 \div 7/4 = 5/6 \times 4/7 = 20/42 = 10/21$. A second way to solve the problem is multiply the entire fraction by the LCD of all the fractions, which is 12: $\frac{12(1/2) + 12(1/3)}{12(2) - 12(1/4)} = \frac{6+4}{24-3} = 10/21$.

6B. Madison drunk 3/7 of the container which means 4/7 of the juice is left over. Now it needs to be divided into 5 cups. Thus, $4/7 \div 5 = 4/7 \div 5/1 = 4/7 \times 1/5 = 4/35$.

7D. Freshmen: 3/8 of $1320 = 3/8 \times 1320 = 495$. Seniors: 2/11 of $1320 = 2/11 \times 1320 = 240$. Adding up the freshmen and seniors you get $495 + 240 = 735$.

8E. In the problem, they are looking for the fraction that is halfway between 1/4 and 8/11. First, add the fractions: $1/4 + 8/11 = 11/44 + 32/44 = 43/44$. Now divide the answer by two: $43/44 \div 2 = 43/44 \div 2/1 = 43/44 \times 1/2 = 43/88$.

9C. The combined fraction of students who were out sick or arrived late was $1/3 + 2/7 = 7/21 + 6/21 = 13/21$. The rest of the students arrived to class on time. Thus, 8/21 of all the students arrived on time.

10B. The smallest fraction will have the smallest numerator (smallest m) and the biggest denominator (biggest p). The smallest value of m is 12 and the biggest value of p is 29. The smallest value of the fraction is 12/29.

11A. Black cars = 4/15 of $60 = 4/15 \times 60 = 16$. White cars = 1/4 of $60 = 1/4 \times 60 = 15$. Gray cars = $7/20 \times 60 = 21$. This means that $16 + 15 + 21 = 52$ cars are either black, white or gray. The rest of the cars ($60 - 52 = 8$ cars) are not those colors.

12A. We will give all the values 5 decimal places:

$0.65599 \rightarrow 0.65599$
$0.\overline{65} \rightarrow 0.65656$
$33/50 \rightarrow 0.66000$
$0.\overline{6} \rightarrow 0.66666$

13D. We will give all the values 5 decimal places:

$234/625 \rightarrow 0.37440$
$0.37\overline{4} \rightarrow 0.37444$
$3/8 \rightarrow 0.37500$
$19/50 \rightarrow 0.38000$
$2/5 \rightarrow 0.40000$

14E. To solve the problem, we can turn the top into a single fraction and then divide by 3.
TOP: $7/3 + 11/6 = 14/6 + 11/6 = 25/6$
Now divide by three: $25/6 \div 3/1 = 25/6 \times 1/3 = 25/18$. A second way to solve the problem is to multiply the entire fraction by the LCD of all the fractions, which is 6. Make sure you change the mixed numbers to improper fractions before multiplying:

$$\frac{6(7/3) + 6(11/6)}{6(3)} = \frac{14 + 11}{18} = 25/18.$$

15D. The combined fraction of boxes that were large or medium were $1/12 + 1/8 = 2/24 + 3/24 = 5/24$. The rest of the boxes were small-sized boxes. Thus, 19/24 of all the boxes were small-sized.

16A. The hard way to do this problem is to use common denominators and subtract the mixed numbers from 4 to find Laura's share of the candy. We will change all the mixed numbers to decimals and compare: John = $1\ 1/4 = 1.25$ pounds, Maxine = $1\ 1/3 = 1.33$ pounds and Laura = $4 - 1.25 - 1.33 \approx 1.42$ pounds. It should now be easier to compare them.

17C. This is a multi-step problem. First find how many pounds of chicken you can buy. You can buy \$11.70 ÷ \$1.30 = 9 pounds of chicken. Each serving is 3/4 of a pound, so we need to divide 9 pounds by 3/4. When you divide, the total comes first and how you split the total comes second (9 is the total pounds and you are splitting into 3/4-pound servings.) Now divide:
$9 \div 3/4 = 9/1 \times 4/3 = 36/3 = 12$ servings.

18A. Number of lawyers = 4/7 of $35 = 4/7 \times 35 = 20$. Now 2/5 of those 20 lawyers are female. The number of female lawyers = 2/5 of $20 = 2/5 \times 20 = 8$.
There are 8 female lawyers in the room.

19B. They want the answer in minutes. Two hours is the same as 120 minutes. Now breakdown the problem: Showering = 1/8 of $120 = 1/8 \times 120 = 15$.
Dinner = 1/4 of $120 = 1/4 \times 120 = 30$.
Homework = 4/15 of $120 = 4/15 \times 120 = 32$.
The total time spent doing those activities were $15 + 30 + 32 = 77$ minutes. The rest of the time that was spent on the phone was $120 - 77 = 43$ minutes.

20E. Changing all the fractions to decimals will show that 3/14 is the smallest fraction and 5/6 is the biggest fraction. Now subtract the fractions:
$5/6 - 3/14 = 35/42 - 9/42 = 26/42 = 13/21$.

21D. We will give all the values 4 decimal places:

$0.\overline{79} \rightarrow 0.7979$
$799/1000 \rightarrow 0.7990$
$0.7\overline{9} \rightarrow 0.7999$
$4/5 \rightarrow 0.8000$

22C. To get the value of X, just add $3\ 2/7$ and $2\ 1/3$ and subtract the answer from $7\ 1/7$. First add the mixed numbers: $23/7 + 7/3 = 69/21 + 49/21 = 118/21$.
Now subtract the answer from $7\ 1/7$: $50/7 - 118/21 = 150/21 - 118/21 = 32/21 = 1\ 11/21$.